

V Barbu (Editor)

Differential equations and control theory

ABOUT THIS VOLUME

This Research Note contains the proceedings of an international conference on differential equations and control theory held at Iași (Romania) in August 1990. Many leading specialists in the field participated in the conference and their contributions reflect the main directions and present state of research. Contributions are focused both on theoretical aspects and applied problems.

Readership: This book will be of interest to specialists in the theory of ordinary differential equations, partial differential equations and control of distributed parameter systems, and to those with an interest in the application of differential equations to mechanics, physics, and biology.

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Parabolic problems associated to integrated circuits with time-dependent sources

We shall be concerned with partial differential systems of the form

$$(S) \quad \frac{\partial v_k}{\partial t} - \frac{\partial}{\partial x}(a_k(x) \frac{\partial v_k}{\partial x}) + g_k(x, v_k) = 0, \quad x \in (0, 1), \quad t > 0 \quad (k = \overline{1, n}),$$

with boundary value conditions

$$(BC) \quad (\gamma_1 v)(t) + G(\gamma_0 v)(t) \ni b(t), \quad t > 0,$$

and initial conditions

$$(IC) \quad v(0, x) = v_0(x), \quad x \in (0, 1),$$

where $v := \text{col}(v_1, \dots, v_n)$, $(\gamma_0 v)(t) := \text{col}(v_1(t, 0), v_1(t, 1), \dots, v_n(t, 0), v_n(t, 1))$,

$$(\gamma_1 v)(t) := \text{col}\left(-a_1(0) \frac{\partial v_1}{\partial x}(t, 0), a_1(1) \frac{\partial v_1}{\partial x}(t, 1), \dots, -a_n(0) \frac{\partial v_n}{\partial x}(t, 0), a_n(1) \frac{\partial v_n}{\partial x}(t, 1)\right).$$

Throughout this article we shall admit the following assumptions:

- (A.1) $a_k \in W^{1,\infty}(0, 1)$ and $a_k(x) > 0$ in $[0, 1]$ ($k = \overline{1, n}$).
- (A.2) $g_k(\cdot, p) \in L_2(0, 1)$ for every $p \in \mathbb{R}$ and $g_k(x, \cdot)$ is continuous and nondecreasing for a.e. $x \in (0, 1)$ ($k = \overline{1, n}$).
- (A.3) $G : D(G) \subset \mathbb{R}^{2n} \rightarrow \mathbb{R}^{2n}$ is a maximal monotone operator (possibly multivalued).

From a physical point of view, problem (S) + (BC) + (IC) is connected with integrated circuit theory. We mention that particular cases of this problem or similar problems were studied in recent years by a Finnish-Romanian group: V. Hara (Jyväskylä), P. Koikkalainen (Jyväskylä), A. Lehtonen (Jyväskylä), C.A. Marinov (Bucharest), G.

Morosanu (Iasi), P. Neittaanmäki (Jyväskylä).

From a theoretical point of view, similar boundary value problems for higher-order parabolic equations were studied by G. Morosanu and D. Petrovanu (see [4, p. 218-245]).

Let us first consider

The case of constant sources: $b(t) \equiv b_0$ (a constant vector).

In this case we can replace G by \tilde{G} defined by $\tilde{G}w = Gw - b_0$, which is also maximal monotone. So we can assume in what follows $b(t) \equiv 0$. This situation was investigated in a previous paper [6] (see also [5], [7]). Let us recall that if $b(t) \equiv 0$, then our problem can be written as a Cauchy problem for an ordinary differential equation in the space $X = L_2(0, 1) \times \dots \times L_2(0, 1)$ (with n factors) endowed with the usual scalar product

$$\langle u, v \rangle_X := \sum_{k=1}^n \langle u_k, v_k \rangle_{L_2(0, 1)} = \sum_{k=1}^n \int_0^1 u_k v_k dx.$$

To this purpose let us define the operator $A : D(A) \subset X \rightarrow X$ as follows:

$$D(A) = \{v \in X; v_k \in H_2(0, 1) \ (k = \overline{1, n}) \text{ and } -\gamma_1 v \in G(\gamma_0 v)\},$$

$$Av := -\text{col} \left(\frac{d}{dx} \left(a_1(\cdot) \frac{dv_1}{dx} \right), \dots, \frac{d}{dx} \left(a_n(\cdot) \frac{dv_n}{dx} \right) \right).$$

Consider also the operator $B : D(B) \subset X \rightarrow X$ defined by

$$Bv := \text{col} (g_1(\cdot, v_1), \dots, g_n(\cdot, v_n)),$$

where $D(B)$ consists of all $v \in X$ such that $Bv \in X$. We know from [6] that (A.1) + (A.3) $\Rightarrow A$ is maximal monotone in X and $D(A)$ is a dense subset of X .

Moreover, (A.1) + (A.2) + (A.30) $\Rightarrow D(A + B) = D(A)$ and $A + B$ is also maximal monotone. If, in addition, G is a subdifferential then $A + B$ is a subdifferential too.

Now, we can see that problem (S) + (BC) + (IC) (with $b(t) \equiv 0$) can be expressed as a Cauchy problem in X :

$$\begin{cases} dv/dt + (A + B)v = 0, & t > 0, \\ v(0) = v_0. \end{cases}$$

We give (without proof) the following existence result:

Theorem 1. Assume that (A.1), (A.2) and (A.3) hold. Then, for every $v_0 \in X$, problem (CP) has a unique weak solution $v \in C(\mathbb{R}_+; X)$. If $v_0 \in D(A)$ then (CP) has a unique strong solution $v \in W^{1,\infty}(0, T; X)$, $\forall T > 0$, with additional properties

$$v_k \in L_\infty(0, T; H_2(0, 1))$$

and

$$v_k, \partial v_k / \partial x \in L_\infty((0, T) \times (0, 1)), \quad \forall T > 0, \quad k = \overline{1, n}.$$

If, in addition G is a subdifferential then for every $v_0 \in X$, v is strong and

$$\sqrt{t} \, dv/dt \in L_2(0, T; X), \quad \forall T > 0.$$

For the definitions of weak and strong solutions for (CP) (hence for (S) + (BC) + (IC)) we refer the reader e.g. to [4, p.47].

Now, we shall concentrate our attention on

The case of time-dependent sources: $b(t) \neq \text{const.}$

We suppose for the time being that $b(t) = \text{col}(b_1(t), \dots, b_{2n}(t))$ is sufficiently regular. Following an idea from [3] we make a change of unknown functions

$$v_k = u_k + \tilde{u}_k \quad (k = \overline{1, n}), \quad (1)$$

where

$$\tilde{u}_k(t, x) = \alpha_k(t)x^3 + \beta_k(t)x^2 + \delta_k(t)x, \quad k = \overline{1, n}, \quad (2)$$

with $\alpha_k, \beta_k, \delta_k$ determined (uniquely) from the system

$$(\gamma_0 \tilde{u})(t) \equiv 0, \quad (\gamma_1 \tilde{u})(t) = b(t). \quad (3)$$

So problem (S) + (BC) + (IC) can be rewritten as

$$\frac{\partial u_k}{\partial t} - \frac{\partial}{\partial x} \left(a_k(x) \frac{\partial u_k}{\partial x} \right) + g_k(x, u_k + \tilde{u}_k(t, x)) h_k(t, x), \quad 0 < x < 1, \quad t > 0, \quad k = \overline{1, n},$$

$$(\gamma_1 u)(t) + G(\gamma_0 u)(t) \ni 0, \quad t > 0. \quad (4)$$

$$u_k(0, x) = u_{k0}(x), \quad 0 < x < 1, \quad k = \overline{1, n},$$

where

$$h_k(t, x) := -\frac{\partial \tilde{u}_k}{\partial t} + \frac{\partial}{\partial x}(a_k(x) \frac{\partial u_k}{\partial x})(t, x),$$

$$u_{k0} := v_{k0}(x) - \tilde{u}_k(0, x).$$

In other words, we have obtained the following *time-dependent* Cauchy problem:

$$\begin{cases} du/dt + Au + B(u + \tilde{u}(t)) = h(t) & \text{in } X, \\ u(0) = u_0. \end{cases} \quad (5)$$

Theorem 2. (Existence of Strong Solutions). Assume that (A.1), (A.2) and (A.3) hold. If $b \in W^{1,2}(0, T; \mathbb{R}^{2n})$ ($T > 0$), $v_{0k} \in H_2(0, 1)$ ($k = \overline{1, n}$ and $b(0) \in \gamma_1 v_0 + G(\gamma_0 v_0)$) then (S) + (BC) + (IC) has a unique strong solution $v \in W^{1,\infty}(0, T; X)$ with additional properties

$$v_k \in L_\infty(0, T; H_2(0, 1)) \text{ and } v_k, \partial v_k / \partial x \in L_\infty((0, T) \times (0, 1)), \quad k = \overline{1, n}. \quad (6)$$

Sketch of proof. In a first stage, we assume that $g_k(x, \cdot)$ are Lipschitz continuous, with Lipschitz constants independent on x , and $b \in W^{2,\infty}(0, T; \mathbb{R}^{2n})$. We consider the operators $Q(t)$, $t \geq 0$ defined by $D(Q(t)) = D(A)$ and $Q(t)u := Au + B(u + \tilde{u}(t)) - h(t)$. Of course, $Q(t)$ are maximal monotone and there exists $L > 0$ such that

$$\|Q(t)u - Q(s)u\|_X \leq L|t - s|, \quad \forall u \in D(A), \quad t, s \in [0, T].$$

Therefore, the family $\{Q(t)\}$ satisfies Kato's conditions (see [1]). On the other hand, we can easily see that $u_0 = v_0 - \tilde{u}(0, \cdot) \in D(A)$. Hence problem (5) has a strong solution u , $u \in W^{1,\infty}(0, T; X)$, $u(t) \in D(A)$ for $t \in [0, T]$. In fact, as $\hat{h}(t) := h(t) - B(u(t) + \tilde{u}(t)) : [0, T] \rightarrow X$ is Lipschitz continuous (hence belongs to $W^{1,1}(0, T; X)$), we have that u is differentiable from the right on $[0, T)$ and

$$\frac{d^+ u}{dt}(t) + Au(t) + B(u(t) + \tilde{u}(t)) = h(t), \quad 0 < t < T,$$

$$u(0) = u_0.$$

Therefore

$$(S) \quad \frac{d^+ v}{dt}(t) + Av(t) + Bv(t) = 0, \quad 0 \leq t < T \text{ in } X,$$

$$(BC) \quad (\gamma_1 v)(t) + G(\gamma_0 v)(t) = b(t), \quad 0 \leq t < T,$$

$$(IC) \quad v(0) = v_0.$$

Now, consider g_k without Lipschitz condition and replace $g_k(x, \cdot)$ by the Yosida approximates $g_{k\lambda}(x, \cdot)$ ($\lambda > 0$). From the reasoning above, problem (5), with $g_{k\lambda}$ instead of g_k , has a strong solution u_λ , i.e. $v_\lambda = u_\lambda + \tilde{u}$ verifies the following problem:

$$\frac{d^+ v_\lambda}{dt}(t) + Av_\lambda(t) + B(\lambda)v_\lambda(t) = 0, \quad 0 \leq t < T \text{ in } X, \quad (7)$$

$$(\gamma_1 v_\lambda)(t) + G(\gamma_0 v_\lambda)(t) = b(t), \quad 0 \leq t < T, \quad (8)$$

$$v_\lambda(0) = v_0. \quad (9)$$

By a standard computation we get

$$\begin{aligned} & \frac{1}{2} \frac{d^+}{dt} \|v_\lambda(t+h) - v_\lambda(t)\|_X^2 + c_0 \left\| \frac{\partial}{\partial x} [v_\lambda(t+h, \cdot) - v_\lambda(t, \cdot)] \right\|_X^2 \\ & \leq \|b(t+h) - b(t)\|_{\mathbb{R}^{2n}} \|(\gamma_0 v_\lambda)(t+h) - (\gamma_0 v_\lambda)(t)\|_{\mathbb{R}^{2n}}, \\ & \quad (0 \leq t < t+h < T, \quad c_0 > 0). \end{aligned}$$

Consequently, taking into account the inequality

$$\|u\|_{C[0,1]} \leq \|u\|_{L_2(0,1)} + \|du/dx\|_{L_2(0,1)}$$

we obtain

$$\begin{aligned} & \frac{d^+}{dt} \|v_\lambda(t+h) - v_\lambda(t)\|_X^2 + 2c_0 \left\| \frac{\partial}{\partial x} [v_\lambda(t+h, \cdot) - v_\lambda(t, \cdot)] \right\|_X^2 \\ & \leq C_1 \|v_\lambda(t+h) - v_\lambda(t)\|_X^2 + C_2 \|b(t+h) - b(t)\|_{\mathbb{R}^{2n}}^2 \\ & \quad + \varepsilon \left\| \frac{\partial}{\partial x} [v_\lambda(t+h, \cdot) - v_\lambda(t, \cdot)] \right\|_X^2 \end{aligned}$$

with $C_1, C_2 > 0$ and $\varepsilon > 0$ small enough.

Hence

$$\begin{aligned} & \frac{d^+}{dt} (e^{-C_1 t} \|v_\lambda(t+h) - v_\lambda(t)\|_X^2 + (2c_0 - \varepsilon)e^{-C_1 t} \left\| \frac{\partial}{\partial x} [v_\lambda(t+h) \right. \\ & \left. - v_\lambda(t)] \right\|_X^2) \leq e^{-C_1 t} C_2 \|b(t+h) - b(t)\|_{\mathbb{R}^{2n}}^2. \end{aligned} \quad (10)$$

This implies

$$e^{-C_1 t} \left\| \frac{d^+ v_\lambda}{dt}(t) \right\|_X \leq \left\| \frac{d^+ v_\lambda}{dt}(0) \right\|_X + C_2 \int_0^t e^{-C_1 r} \|b'(r)\|_{\mathbb{R}^{2n}}^2 dr, \quad 0 \leq t < T. \quad (11)$$

On the other hand, we can easily see that

$$\sup_{\lambda > 0} \left\| \frac{d v_\lambda}{dt}(0) \right\|_X < \infty. \quad (12)$$

From (11) and (12) we get

$$\{dv_\lambda/dt; \lambda > 0\} \text{ is bounded in } L_\infty(0, T; X) \quad (13)$$

and hence

$$\{v_\lambda; \lambda > 0\} \text{ is bounded in } L_\infty(0, T; X). \quad (14)$$

Now denote $g_\lambda := -Av_\lambda - B(\lambda)v_0$ and remark that $\{g_\lambda; \lambda > 0\}$ is bounded in X . Multiplying the equation

$$\frac{d}{dt} (v_\lambda - v_0) + Av_\lambda - Av_0 + B(\lambda)v_\lambda - B(\lambda)v_0 = g_\lambda$$

by $v_\lambda - v_0$ we can obtain an estimate similar to (10)

$$\begin{aligned} & \frac{d^+}{dt} (e^{-C_3 t} \|v_\lambda(t) - v_0\|_X^2) + (2c_0 - \varepsilon)e^{-C_3 t} \left\| \frac{\partial}{\partial x} [v_\lambda(t, \cdot) - v_0(\cdot)] \right\|_X^2 \\ & \leq \text{const}, \quad 0 \leq t < T. \end{aligned} \quad (15)$$

From (13), (14) and (15) we get

$$\left\{ \frac{\partial v_\lambda}{\partial x}; \lambda > 0 \right\} \text{ bounded in } L_\infty(0, T; X). \quad (16)$$

Using (14), (16) and the formula

$$v_{k\lambda}(t, x) = \int_0^1 (y \frac{\partial v_{k\lambda}}{\partial y}(t, y) + v_k(t, y)) dy - \int_x^1 \frac{\partial v_{k\lambda}}{\partial y}(t, y) dy,$$

we can see that

$$\{v_{k\lambda}; \lambda > 0\} \text{ is bounded in } L_\infty((0, T) \times (0, 1)), \quad k = \overline{1, n}. \quad (17)$$

This implies

$$\{B(\lambda)v_\lambda; \lambda > 0\} \text{ is bounded in } L_2(0, T; X). \quad (18)$$

Now, from the obvious inequality

$$\frac{1}{2} \frac{d}{dt} \|v_\lambda(t) - v_\mu(t)\|_X^2 \leq -\langle B(\lambda)v_\lambda - B(\mu)v_\mu, v_\lambda - v_\mu \rangle_X,$$

we deduce that

$$\|v_\lambda(t) - v_\mu(t)\|_X \leq C_4(\lambda + \mu)^{1/2}, \quad 0 \leq t \leq T,$$

and this shows that v_λ converges to some v in $C([0, T]; X)$, as $\lambda \rightarrow 0 \Rightarrow u_\lambda = v_\lambda - \tilde{u} \rightarrow u = v - \tilde{u}$ in $C([0, T]; X)$. Next, by Lebesgue's Dominated Convergence Theorem, we can prove that $B(\lambda)v_\lambda \rightarrow Bv$, strongly in $L_2(0, T; X)$. Now we are able to pass to the limit in the equation

$$dv_\lambda/dt + Av_\lambda + B(\lambda)v_\lambda = 0,$$

to obtain that v is a strong solution of (S) + (BC) + (IC).

In the next step, we shall consider that $b \in W^{1,2}$ (instead of $b \in W^{2,\infty}$). Taking v_0, \bar{v}_0 such that $v_0 - \tilde{u}(0, \cdot), \bar{v}_0 - \tilde{u}(0, \cdot) \in D(A)$ and $b, \bar{b} \in W^{2,\infty}$ we can show (after some computations) that the corresponding solutions v, \bar{v} satisfy

$$\|v(t) - \bar{v}(t)\|_X^2 \leq \text{const.} \left(\|v_0 - \bar{v}_0\|_X^2 + \int_0^t \|b(s) - \bar{b}(s)\|_{\mathbb{R}^{2n}}^2 ds \right), \quad 0 \leq t \leq T, \quad (19)$$

and

$$\|v(t+h) - v(t)\|_X^2 \leq \text{const} \left(\|v(h) - v_0\|_X^2 + \int_0^t \|b(s+h) - b(s)\|_{\mathbb{R}^{2n}}^2 ds \right),$$

$$0 \leq t < t+h \leq T. \quad (20)$$

Now, taking a sequence $\{b_m\} \subset W^{2,\infty}$ such that $b_m \rightarrow b$ in $W^{1,2}$ and fixing v_0 such that $u_0 = v_0 - \tilde{u}(0, \cdot) \in D(A)$ we can see that the corresponding sequence of strong solutions v_m converges uniformly to some v which is also a strong solution. The regularity properties (6) follow by standard reasonings. Q.E.D.

Theorem 3. (Existence of Weak Solutions). If (A.1), (A.2), (A.3) hold, $v_0 \in X$ and $b \in L^2(0, T; \mathbb{R}^{2n})$, then (S) + (BC) + (IC) has a unique weak solution $v \in C([0, T]; X)$ with $v_k \in L_2(0, T; W^{1,2}(0, 1))$ ($k = \overline{1, n}$).

Proof. Let $\{v_0(j)\}_{j \geq 1} \subset D(A)$ be such that $v_0(j) \rightarrow v_0$ in X and let $\{b(j)\}_{j \geq 1} \subset W^{1,2}(0, T; \mathbb{R}^{2n})$ be such that $b(j)(0) = 0$ and $b(j) \rightarrow b$ in L_2 . Then, the corresponding strong solutions $v(j)$ satisfy

$$\|v(j)(t) - v(i)(t)\|_X + \int_0^t \left\| \frac{\partial}{\partial x} [v(j)(s) - v(i)(s)] \right\|_X^2 ds$$

$$\leq \text{Const} \int_0^t \|b(j)(s) - b(i)(s)\|_{\mathbb{R}^{2n}}^2 ds, \quad 0 \leq t \leq T,$$

which leads us to the conclusion. Q.E.D.

Theorem 4. (Asymptotic Behaviour of Solutions). If (A.1), (A.2), (A.3) hold, G is strongly monotone and $b \in L_2(\mathbb{R}_+; \mathbb{R}^{2n})$, then $A+B$ is strongly monotone, hence $(A+B)^{-1} 0$ has one element, say p , and $v(t) \rightarrow p$ in X as $t \rightarrow \infty$, for every weak solution $v(t)$.

Proof. An easy computation shows that $A+B$ is indeed strongly monotone. Now for each $j \in \mathbb{N}$ we define $b(j)$ by

$$b(j)(t) = \begin{cases} b(t), & 0 \leq t \leq j, \\ 0, & t > j, \end{cases}$$

and denote by $v(j)(t)$, $t \geq 0$, the solutions corresponding to $b(j)$ and satisfying

$v(j)(0) = v(0)$. Note that

$$\|v(j)(t) - v(t)\| \leq \text{const} \left(\int_j^\infty \|b(s)\|_{\mathbb{R}^{2n}}^2 ds \right)^{1/2}. \quad (21)$$

Since for $t \geq j$, $v(j)(t)$ is a solution corresponding to null sources we have $v(j)(t) \rightarrow p$, strongly in X as $t \rightarrow \infty$. This fact combined with (21) and with the following inequality

$$\|v(t) - p\| \leq \|v(t) - v(j)(t)\| + \|v(j)(t) - p\|$$

implies that $v(t) \rightarrow p$, as $t \rightarrow \infty$.

Q.E.D.

Remark. For other details and for more general problems we refer the reader to [2] and [5].

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