relation to the likelihood of finding this evidence if any other plausible explanation was valid (p(e|~h)). Bayesian reasoning gives us a helpful language for describing the research process where we update our confidence in the validity of a given theory based on the strength of empirical tests that we are able to deploy.

However, as qualitative scholars we do not believe that we possess the type of data that would enable us to make "objective" predictions regarding prior probabilities and likelihood ratios that quantitative Bayesian statisticians can make, which is why we chose not to engage with this literature—especially given that our audience is qualitative case study scholars and not quantitative statisticians. Instead, we refer our readers to discussions about Bayesian reasoning and subjectivity to Howson and Urbach (2006: 265–272) and Chalmers (1999: 177–192). Here we pragmatically suggested, as many social scientists before us, that Bayesian reasoning focuses our attention on what elements our tests need to include that we need to evaluate qualitatively in order to make as strong causal inferences as possible. This is why many process-tracing methodologists either explicitly (Bennett 2008a) or implicitly (Mahoney 2012) refer to Bayesian reasoning.

References

Bennett, Andrew. 2008a. "Process Tracing: A Bayesian Perspective." In *The Oxford Handbook of Political Methodology*. Janet Box-Steffensmeier, Henry E. Brady and Collier David, eds. (Oxford: Oxford University Press), 702–721.

Bennett, Andrew. 2008b. "The Mother of All 'Isms': Organizing Political Science around Causal Mechanisms." In *Revitalizing Causality: Realism about Causality in Philosophy and Social Science*. Ruth Groff, ed. (London: Routledge), 205–219.

Bennett, Andrew and Jeffrey Checkel. Forthcoming. "Introduction." *Process Tracing in the Social Sciences: From Metaphor to Analytic Tool*. Cambridge: Cambridge University Press.

Chalmers, Alan. F. 1999. *What is this Thing Called Science?* Buckingham: Open University Press.

Evans, Peter. 1995. "Contribution to Symposium on The Role of Theory in Comparative Politics." *World Politics* 48:1, 3–10.

George, Alexander L. and Andrew Bennett. 2005. *Case Studies and Theory Development in the Social Sciences*. Cambridge: MIT Press.

Howson, Colin and Peter Urbach. 2006. *Scientific Reasoning: The Bayesian Approach*. Third Edition. La Salle, Il: Open Court.

Machamer, Peter. 2004. "Activities and Causation: The Metaphysics and Epistemology of Mechanisms." *International Studies in the Philosophy of Science* 18:1 (March), 27–39.

Mahoney, James. 2012. "The Logic of Process Tracing Tests in the Social Sciences." *Sociological Methods and Research* 41:4, 570–597.

Waldner, David. 2012. "Process Tracing and Causal Mechanisms." In *Oxford Handbook of the Philosophy of Social Science*. Harold Kincaid, ed. (Oxford: Oxford University Press), 65–84.

Fuzzy Sets are Sets—A Reply to Goertz and Mahoney

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The Common Ground

As stated in our first comments, we share G&M's vision of qualitative research as being rooted in set theory. Precisely because we think that this is a plausible proposition with potentially even more fruitful and intriguing implications than mentioned in G&M's book, we expressed some concern as to whether or not G&M will get their message through in the way their argument is formulated. By and large, our apprehension is triggered by a similar observation formulated by Elman (2013) who argues that some of G&M's propositions about the set-theoretic nature of qualitative research are prescriptive while others are descriptive. We believe that in the discussions to follow, more clarity in distinguishing between what qualitative research currently is and what it ought to be would help to strengthen G&M's vision of qualitative research as mainly being set-theoretic.

We take up G&M's quest and focus this round of debate more on issues that arise if and when one accepts set theory as the foundational system of thought of qualitative research. Above all, we respond to G&M's claim that fuzzy sets do not establish qualitative differences between cases that fall above and below the fuzzy set membership score of 0.5. In addition, we provide two examples, going beyond G&M, of how the notions of sets and their relations can be used to inform standard issues in qualitative social research: case selection in multi-method research and theory evaluation.

Qualitative Thresholds in Fuzzy Sets

G&M challenge the claim made by us and others in the field (e.g., Ragin 2000) that fuzzy sets, first and foremost, establish qualitative differences between cases ('differences in kind') and that this difference is established by whether a case holds a fuzzy set membership of higher or lower than 0.5 in a given fuzzy set. They, instead, find it "awkward and misleading" to interpret the difference between a case with a fuzzy value of 0.49 and another one with 0.51 as a qualitative one. For them, this difference is only a matter of degree with no particular relevance for the conceptual status of the cases in question. They claim that our insistence on a qualitative difference takes away the advantage of fuzzy sets, namely to recognize nuances. In the following, we provide arguments why we think G&M's position is either a misunderstanding of our claim or leads to untenable consequences.

First, and perhaps most importantly, all sets, by their very definition, establish a qualitative distinction between those cases that are members and those that are not. This also applies to fuzzy sets; otherwise, they would not be sets. The only information fuzzy sets add to this qualitative distinction is that cases can be ranked as to how much they are (not) members of a set. As a matter of fact, we think that much of the plausibility of G&M's postulate "qualitative research = set-theoretic research" precisely stems from the fact that sets reflect qualitative properties of cases and qualitative researchers are interested in, well, the cases' qualitative properties and their relations. Consider the Latin origin of the words "qualitative" and "quantitative," respectively. *Quantum* is translated as "how many" or "how much." Objects are assessed as to how much of a certain property they possess. The word *qualis*, instead, is translated as "what" or "of what kind/sort/nature." Qualitative statements about objects establish *qualitative* differences between objects that are in a given set and and those that are out. We deem this property of sets both uncontroversial and constitutive for the vision of qualitative research as being rooted in set theory.

With crisp sets, all this seems pretty uncontroversial. Cases have either full membership or full non-membership, and the Rule of the Excluded Middle conveniently rules out any confusion between these two mutually exclusive and jointly exhaustive qualitative states in which cases can be. What seems confusing is whether or not the properties of fuzzy sets—partial membership scores and the breakdown of the Rule of the Excluded Middle—unavoidably mean that (a) qualitative distinctions between cases cannot be made; and, relatedly, that (b) one and the same case should be allowed to hold high membership in two or more mutually exclusive sets.

We believe that (a) fuzzy sets first and foremost express which cases are qualitatively identical and different, and only afterwards to which degree cases are members of a given set; and (b) that cases cannot simultaneously be good empirical instances of two or more sets that have been created precisely because researchers believe that there are two or more qualitatively distinct phenomena worth being given different names. Applied to G&M's example, if we believe that democracies are not only qualitatively different from autocracies, but also from anocracies, and we therefore introduce this new term into the literature, then it seems confusing to us that the exact *same* empirical property of a case (e.g. a Polity score of 7) qualifies a case as being a good instance of two conceptually distinct sets (e.g., democracy and anocracy).

We are not sure how exactly G&M position themselves on these two issues. They do argue that partial membership in two or more mutually exclusive sets should be possible and they have no issue with such partial membership being higher than 0.5 in more than one set. We agree that partial membership in mutually exclusive sets is unproblematic—if and only if, however, a case's membership does not exceed 0.5 in more than one of these sets. This is where we disagree with G&M. This allows two interpretations. G&M might claim that fuzzy sets do not establish any qualitative distinctions and that all is just a matter of degree. For reasons outlined above, this runs

so much counter to the notion of sets and the whole logic of their book that we dismiss this as a plausible interpretation of their argument.

Alternatively G&M might claim that the qualitative distinction is not made by the 0.5 but some other membership score. There are two candidates. One might want to claim that only full members of a set (1) are qualitatively different from the rest. Or one might claim that only full non-members (0) are qualitatively different from the rest. Both options seem problematic to us. Claiming that only cases with full membership in the set of, say, democracy should count as democracies often means that most analyses are exclusively performed on non-democracies, for often times there are no ideal-typical instances of democracy in the data. Likewise, using 0 membership as the qualitative threshold would often mean that researchers exclusively have democracies in their data. While these arguments run against 0 or 1 as qualitative anchors, the following provides an argument in favor of this anchor being at 0.5. By pure mathematical necessity, one and the same case can hold membership in one, and only one, of all the logically possible conjunctions between two or more sets. Although one and the same case will have fuzzy set membership scores of higher than 0 in most, and often even all, of the logically possible types, it can be exclusively attributed to only one of the mutually exclusive and jointly exhaustive ideal types—if and when 0.5 is interpreted as the qualitative anchor. This property of fuzzy sets is constitutive for the so-called fuzzy-set truth table algorithm (Ragin 2008), which, in turn, is at the heart of virtually all applied Qualitative Comparative Analysis.

It should also be noted that regardless of where G&M believe the qualitative threshold is located (and it must, we contend, be somewhere), one of their arguments against 0.5 as a qualitative anchor is not tenable, namely that small differences in fuzzy set membership (e.g., 0.49 vs. 0.51) should not establish qualitative differences. The same small difference in set membership scores of 0.02 would also establish a qualitative difference if the threshold was at 0 or at 1. If we accept that sets establish qualitative differences and thus that there must be a threshold, then differences between two fuzzy values have to be interpreted in different manners, depending on whether the two values are on the same side of the threshold or on two different sides.

Saying that fuzzy sets first and foremost establish qualitative distinctions does not mean that researchers lose sight of differences in degree. Put differently, saying that a case with 0.51 in the set of democracies should count as a democracy while another case with 0.49 should not does not prevent a researcher from seeing that the 0.51 case is much more ambiguous than a third case with a membership of, say, 0.91, which is a much clearer empirical manifestation of the ideal typical democracy. This should make clear why 0.5 is also sometimes called "the point of maximum ambiguity (i.e., fuzziness)" (Ragin 2008: 30). Note as well that this information on differences in degree among qualitatively identical cases never gets lost in applied fuzzy set social science, not even in QCA, which, as mentioned, crucially rests on the qualitative distinction imposed by the 0.5 anchor. For instance, all the formulas for

Figure 1: Wavelength and Membership in Types of Color
Source: Lakoff (1987: 27)
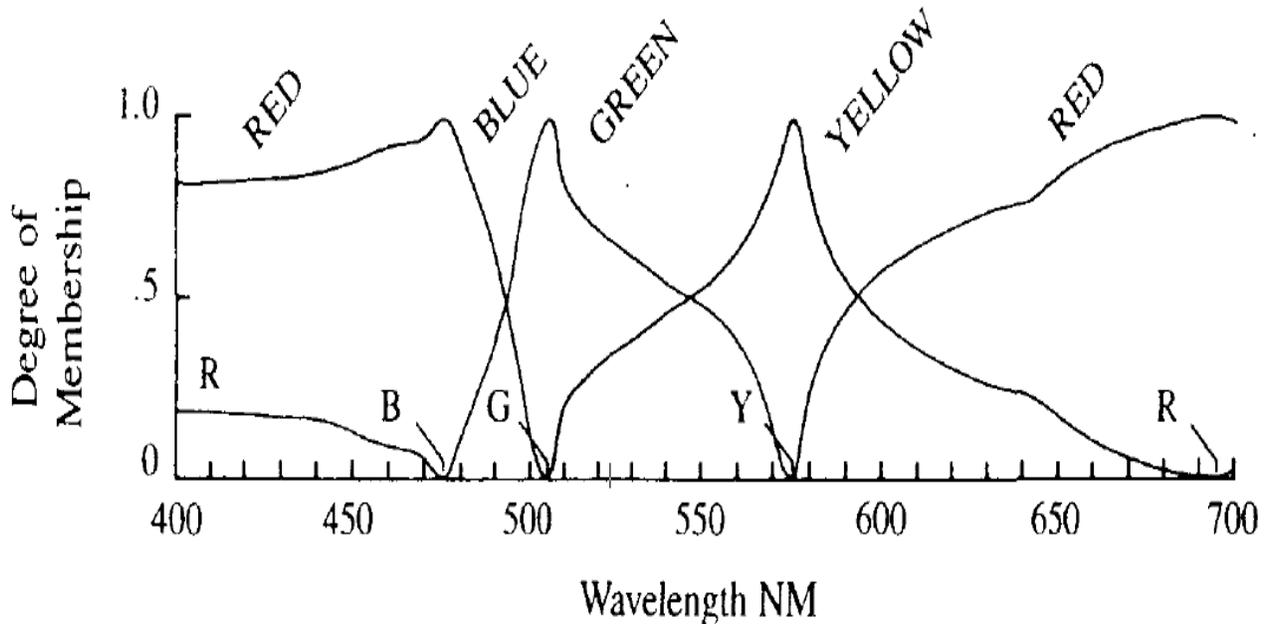
calculating the parameters of fit for set-relational statements make use of the fuzzy set membership scores of each case in all sets involved.

Last but not least, the use of 0.5 as a cutoff for qualitative distinction does not seem to be a peculiarity of social scientists using fuzzy sets. For illustration, consider Figure 1 borrowed from Lakoff's (1987) book, *Women, Fire, and Dangerous Things*. Wavelength is the empirical information used for calibrating the fuzzy sets of different types of spectral colors— just as Polity scores are used for calibrating membership in different types of political regimes. Sure enough, any given wavelength produces partial membership in more than one type of color. Yet, no single wavelength yields a membership of higher than 0.5 in more than one type of color. At worst, a given wavelength produces maximum ambiguity by yielding fuzzy set membership scores of exactly 0.5 in two color sets (e.g., 590 NM as 0.5 fuzzy membership in both yellow and red). Confronted with such ambiguous classification, researchers— or language groups for that matter—might decide to introduce a new type of color, say "orange," and define a case with 590 MN wavelength as having above 0.5 membership in orange and below 0.5 in both red and yellow—just as researchers introduce anocracy as a new type of political regime for those cases that otherwise would have an ambiguous status vis-à-vis already existing types of regime.

A Set-Theoretic Perspective on Case Selection and Theory Evaluation

No doubt, the range of topics addressed by G&M is breathtaking. Yet, looking at social science research from a set-theoretic angle promises to lay the ground for an even more extensive methodological agenda. We highlight only two such

issues that remain underdeveloped in G&Ms book and the literature at large: set-theoretic multi-method research (MMR) and set-theoretic theory evaluation.[1]

MMR is *en vogue*. Yet, the literature is only slowly responding to the task of formulating principles and practices of one specific form of MMR: a set-theoretic cross-case analysis (read: QCA) followed by within-case analysis. The emerging literature on this topic (Rohlfing and Schneider 2013; Schneider and Rohlfing 2013) identifies different types of cases, some of which are similar to regression-based multi-method research (e.g. typical cases), while others are different (e.g., irrelevant cases, deviant cases consistency, or deviant cases coverage). The distinction between these types crucially rests on their membership above or below the 0.5 qualitative anchor in the condition and outcome sets. At the same time, this literature also proposes mathematical formulas for identifying the *best available* cases for each of these types and these formulas rely on the degree of membership of cases. This, incidentally, provides further evidence that fuzzy sets can do both: establishing qualitative differences and expressing differences in degree. Since set-theoretic MMR principles and practices are decidedly different from the better-known regression-based MMR, further research needs to go into how to perform this type of MMR.

The same also holds for another topic often linked to qualitative research: theory evaluation (as opposed to theory testing, which is at the heart of most quantitative research). Early on, Ragin (1987: 118–121) alerted researchers to the possibility of using Boolean algebra for "evaluating theoretical arguments" derived by set-theoretic methods. In essence, researchers present both their theoretical hunches (T) and their empirical findings (E) in the form of Boolean expressions. The intersec-

tion between T and E (TE) yields a Boolean expression that describes where theory and empirics overlap (i.e., this is the part of the theoretical expectations that is supported by empirical evidence). The intersection between T and ~E (T~E), instead, yields a Boolean expression of which part of the theory is not supported by the empirical findings; and ~TE, in turn, is where the empirical analysis revealed conjunctural causes for the outcome not foreseen by the theory.

This basic template of theory evaluation needs to be further refined and made compatible with recent developments in QCA. Most importantly, virtually no applied QCA yields fully consistent results, nor are always all instances of the outcome explained, or covered. In such a scenario, the different intersections between theory and empirical findings sketched out above will have different meanings depending on whether they yield Boolean expressions that mostly describe cases that are members of the outcome under study or non-members thereof. In our book (Schneider and Wagemann 2012: 295–305), we make a first attempt at spelling out this extended and, admittedly, more complicated and intricate version of set-theoretic theory evaluation. More research has to be done. For instance, much of what we write resonates with well-established notions such as "least likely" or "most likely" cases and the guidance they provide for case selection. In addition, more would need to be written on how theory evaluation is influenced by the common QCA practice of making assumptions on so-called logical remainders, and here especially easy counterfactuals.

In sum, we agree with G&M that an explicit set-theoretic take on social science research opens an exciting and largely unexplored research agenda, and have sketched out some details of that agenda. This agenda should not be dismissed simply because not everybody would like to, nor needs to, be subsumed under a set-theoretic perspective.

Note

[1] We address these and other issues, such as "robustness tests in set-theoretic methods," more extensively in our book (Schneider and Wagemann 2012).

References

Elman, Colin. 2013. "Duck-Rabbits in Social Analysis: A Tale of Two Cultures." *Comparative Political Studies* 46:2, 266–277.

Lakoff, George. 1987. *Women, Fire, and Dangerous Things: What Categories Reveal About the Human Mind*. Chicago: University of Chicago Press.

Ragin, Charles C. 1987. *The Comparative Method: Moving Beyond Qualitative and Quantitative Strategies*. Berkeley: University of California Press.

Ragin, Charles C. 2000. *Fuzzy-Set Social Science*. Chicago: University of Chicago Press.

Ragin, Charles C. 2008. *Redesigning Social Inquiry: Fuzzy Sets and Beyond*. Chicago: University of Chicago Press.

Rohlfing, Ingo and Carsten Q. Schneider. 2013. "Combining QCA with Process Tracing in Analyses of Necessity." *Political Research Quarterly* 66:1, 220–235.

Schneider, Carsten Q. and Ingo Rohlfing. 2013. "Set-Theoretic Methods and Process Tracing in Multi-Method Research: Principles of Case Selection After QCA." *Sociological Methods and Research*, forthcoming.

Schneider, Carsten Q. and Claudius Wagemann. 2012. *Set-Theoretic Methods for the Social Sciences: A Guide to Qualitative Comparative Analysis*. Cambridge: Cambridge University Press.

Battle of the Books

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We are grateful for the thoughtful discussion of our book by Goetz and Mahoney and the valuable remarks of other contributors. In our final statement we want to address three points. First, we would like to revisit our distinction between three approaches to explanatory case studies. This is triggered by Goertz and Mahoney's statement that our congruence approach usually requires elements of one of our two other approaches, the covariational approach and causal-process tracing. Secondly, in a response to comments by Rohlfing we want to briefly clarify what we mean by a co-variational approach to explanatory small-N research. Finally, we would like to take up the notion of Goertz and Mahoney that we are witnessing a third wave of qualitative methodology. We will argue that work of this new wave is particularly promising when it comes to closing the gap between those interested in the methodological per se and those looking for concrete guidelines and advice for actually conducting qualitative research.

Goertz and Mahoney address our distinction between three approaches to explanatory case study research. They argue that to achieve the goal of congruence analysis, which is the evaluation of the explanatory power of alternative theories, we usually need to draw either on the covariational approach or on causal process tracing. We would like to stress that we take a broader and more pluralistic epistemological perspective on theory development in the social sciences than the other books and this implies that congruence analysis is not just a mix between the other two approaches.

When dealing with theories in the context of congruence analysis, we have theories in mind that operate on a higher level of abstractions than theories (or explanations) that are typically utilized in research adhering to the co-variational approach and or causal-process tracing. These theories are often explicitly linked to paradigms, they are not oriented towards a specified population, and they often embody propositions about constitutive concepts. Such theories are for instance prominent in International Relations (think of rationalism and social constructivism) and they often link empirical research to political philosophy.

The co-variational approach has more affinities with what could be called "empiricist" theories. This understanding of theories fits the view presented in the books of Goertz and